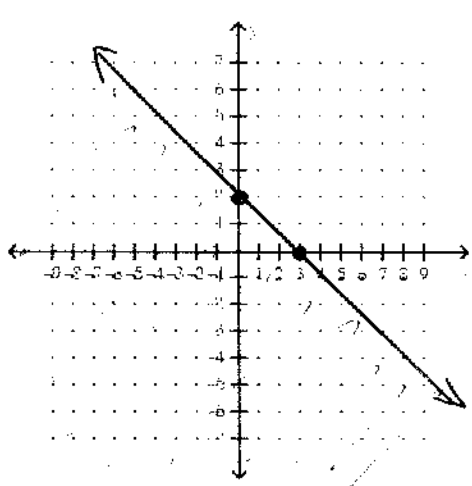
Lesson Objective: Students will be able to graph systems of inequalities and use linear programming to maximize profits or minimize costs for a design product.

Review of Graphing Linear Inequalities:
There are 2 ways to graph lines:
1. Re-write in slope intercept form (y = mx + b) 2. Solve for the x- and y-intercept.

EX: $4x + 6y \le 12$

Slope-intercept y=mx+b form	Standard form ax+by=c, use x & y intercepts
1. $4x + 6y \le 12$ $\underline{-4x \qquad -4x}$ $\frac{6y}{6} \le \frac{-4x + 12}{6}$ $y \le -\frac{2}{3}x + 2$ To GRAPH: 1. Begin with b (y-intercept), plot point at 2 on the y-axis 2. From the y-intercept, use the slope $\frac{-2}{3}$ and go DOWN 2, RIGHT 3, plot the 2nd point. 3. Draw the line 4. Pick a test point, like (0,0) to substitute into the inequality to determine if it is TRUE OR FALSE. If TRUE, shade the region where the test point is located on the graph. If false, shade the region where the test point is NOT located on the graph.	2. $4x + 6y \le 12$ <u>x-intercept:</u> Let y =0 and solve for x. <u>y-intercept:</u> Let x = 0 and solve for y. $4x + 6(0) = 12$ $\quad$ $4(0) + 6y = 12$ $\frac{4x}{4} = \frac{12}{4}$ $\quad$ $\frac{6y}{6} = \frac{12}{6}$ x = 3 $\quad$ y = 2 (3, 0) $\quad$ (0, 2) 3. Plot the points on your graph and connect the dots. 4.Pick a test point, like (0,0) to substitute into the inequality to determine if it is TRUE OR FALSE. If TRUE, shade the region where the test point is located on the graph. If false, shade the region where the test point is NOT located on the graph.

Graph of **$4x + 6y \le 12$**



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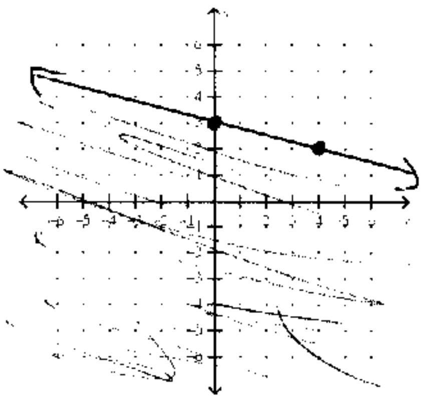
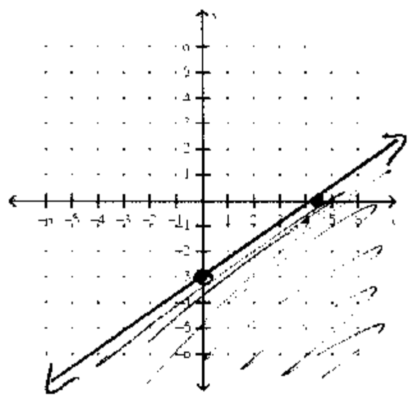
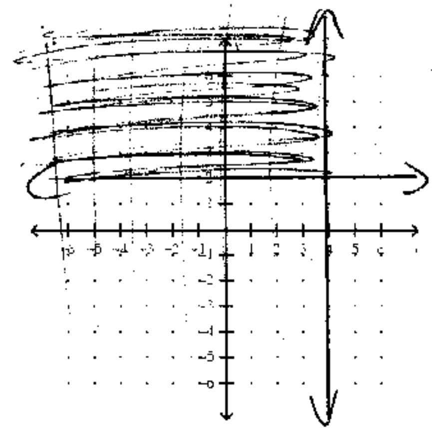
Try it: Graph the inequalities

$y \leq \frac{-1}{4}x + 3$ $\quad$ $2x - 3y \geq 9$ $\quad$ $y \geq 2 \; and \; x \leq 4$







How can graphic artists/printers determine how much to charge a customer to maximize their profits and minimize costs? What do they need to know to determine this?

Linear Programming: a technique that involves maximizing or minimizing an objective function by subjecting it to constraints.

The constraints are a system of linear inequalities, which are graphed to determine a set of feasible solutions from the boundaries (vertices) of the graph. These sets of possible solutions are then entered into the objective function to determine the maximum or minimum.

Steps to solve a Linear Programming Problem:
1) Graph the region that satisfies all of the constraints.
2) Find the intersection points that bound the region, also known as the feasible solutions.
3) Evaluate the objective function at each vertex of the region.
4) Identify the solution that gives the maximum or minimum value of the function.

Example: Maximize profit

A screen printer has no more than $750 to spend on printing T-shirts and sweatshirts for a 5-K charity race. He must print at least 45 T-shirts and 10 sweatshirts. The supplies and labor cost $5 per T-shirt shirt and $15 per sweatshirt. The screen printer will make a profit of $12 per T-shirt and $35 per sweatshirt. How many of each should be printed to maximize the profit? What is the maximum profit?

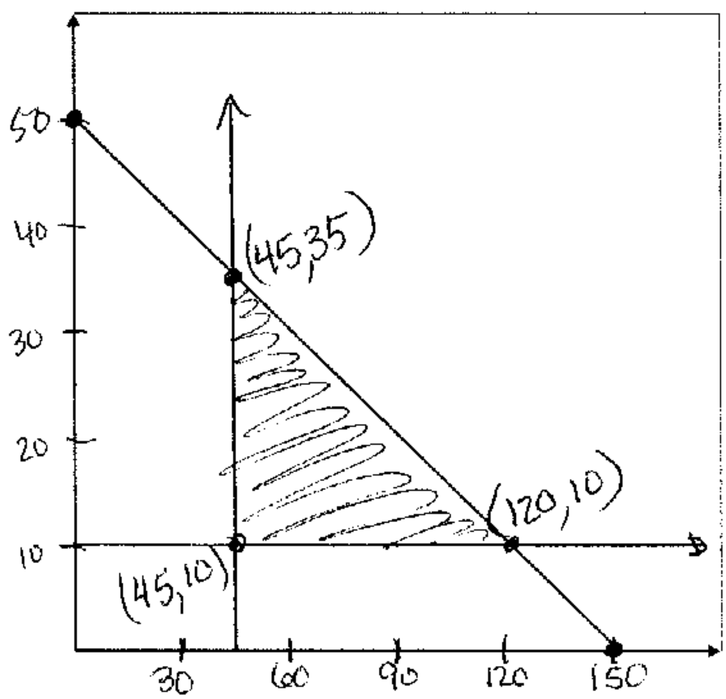
x = T-shirt
y = Sweatshirt

Profit = 12x + 35y

Constraints:
$x \geq 45, y \geq 10$
$5x + 15y \leq 750$ (0, 50) (150, 0)

Steps to solve:

1) Identify variables, x and y
2) Write an equation representing cost
3) Write constraints as inequalities
4) Graph constraints, and find points that bound the feasibility (shaded) area
5) Substitute the points into the cost equation to determine the values of x and y that minimize the cost



Vertices	P = 12x +35y
(45, 10)	890
(120, 10)	1790
(45, 35)	1765

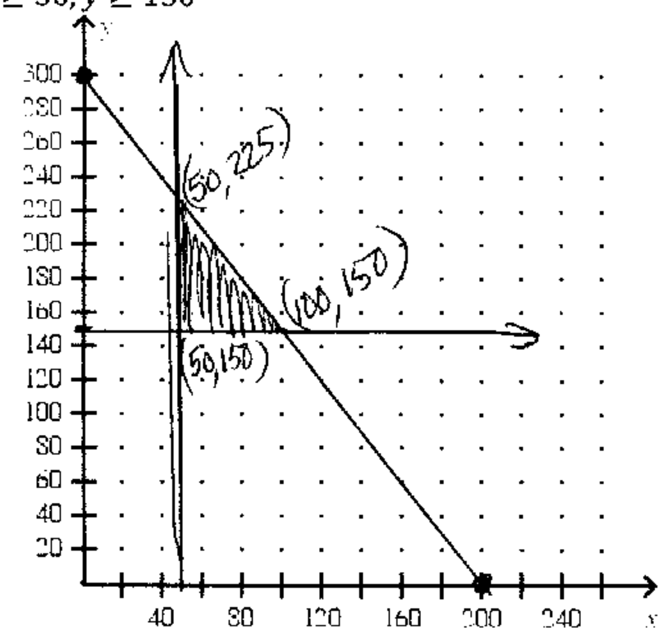
A maximum profit of $1790 will be made by printing 120 T-shirts and 10 sweatshirts.

Example: Minimize cost
A printer is hired to printer brochures and fliers for a travel agency to advertise special discounts and vacation destinations for spring break. Each brochure costs $.08 to print, and each flier costs $.04 to print. A brochure uses 3 pieces of paper, and a flier uses 2 pieces of paper. The printer does not want to use more than 600 pieces of paper, and she needs at least 50 brochures and 150 fliers. How many of each should she print to minimize the cost? What is the minimum cost?

x = brochure
y = flier

Cost = .08x + .04y

Constraints:
$3x + 2y \leq 600$ (0,300) (200,0)
$x \geq 50, y \geq 150$



Steps to solve:

1) Identify variables, x and y
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Vertices	C = .08x + .04y
(50, 150)	10
(50, 225)	13
(100, 150)	14

50 brochures and 150 fliers will minimize the cost.
Minimum cost = $10

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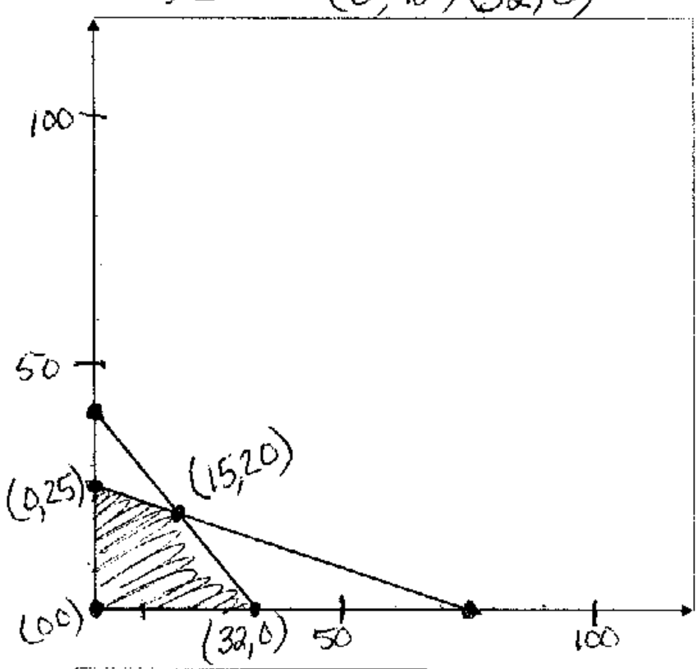
Try it: A factory makes breakfast bars and granola bars. The factory makes a $40 profit from each case of breakfast bars and a $55 profit from each case of granola bars. It takes 2 machine hours and 5 hours of labor for each case of breakfast bars and 6 machine hours and 4 hours of labor for each case of granola bars. The factory has a maximum of 150 machine hours and 160 labor hours available. How many cases of each product should be produced in order to maximize the profit? What is the maximum profit?

X = breakfast bars
Y = granola bars

Profit = 40x + 55y

Constraints:
$x \geq 0, y \geq 0$
$2x + 6y \leq 150$ (0,25) (75,0)
$5x + 4y \leq 160$ (0,40) (32,0)



Vertices	P = 40x + 55y
(0, 0)	0
(0, 25)	$1375
(15, 20)	$1700
(32, 0)	$1280

x= 15 cases of breakfast bars
y=20 cases of granola bars
Maximum Profit = $1700

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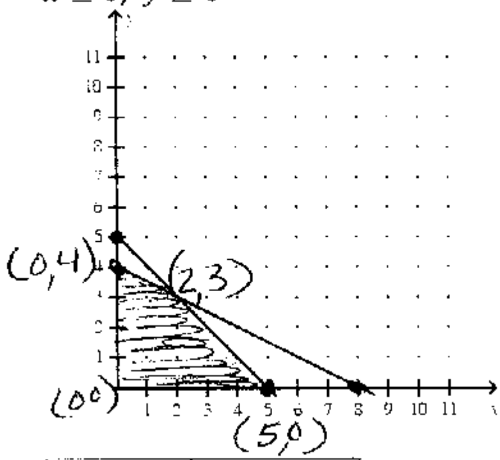
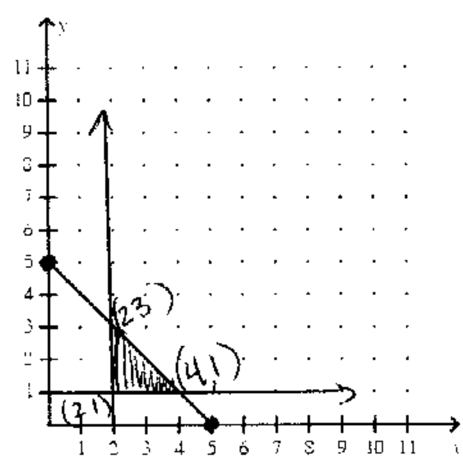
Example 3:
If profit is P = x + 3y, find the maximum profit under the following constraints:
$x + y \leq 5$
$x + 2y \leq 8$
$x \geq 0,\ y \geq 0$



Vertices	P = x + 3y
(0, 0)	0
(0, 4)	12
(2, 3)	11
(5, 0)	5

Maximum profit of 12 at (0, 4)

Try it:
If cost is C = 2x + 3y, find the minimum cost under the following constraints:
$x + y \leq 5$
$x \geq 2,\ y \geq 1$



Vertices	C = 2x + 3y
(2, 1)	7
(2, 3)	13
(4, 1)	11

Minimum cost of 7 at (2, 1)