Instructor Notes Maximizing Profits

Lesson Objective: Students will be able to graph systems of inequalities and use linear programming to maximize profits or minimize costs for an agricultural product..

Review of Graphing Linear Inequalities:

There are 2 ways to graph lines:

1. Re-write in slope intercept form ($y = mx + b$)

2. Solve for the x- and y-intercept.

EX: $4x + 6y \le 12$

<table>
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<td>
Slope-intercept y=mx+b form
1. $4x + 6y \le 12$
$\underline{-4x \qquad -4x}$
$\dfrac{6y}{6} \le \dfrac{-4x + 12}{6}$
$y < -\dfrac{2}{3}x + 2$
To GRAPH:
1. Begin with b (y-intercept). plot point at 2 on the y-axis

2. From the y-intercept, use the slope $\left(-\dfrac{2}{3}\right)$ and go DOWN 2, RIGHT 3, plot the 2nd point.
3. Draw the line
4. Pick a test point, like (0,0) to substitute into the inequality to determine if it is TRUE OR FALSE. If TRUE, shade the region where the test point is located on the graph. If false, shade the region where the test point is NOT located on the graph.
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<td>
Standard form ax+by=c, use x & y intercepts
2. $4x + 6y \le 12$
<u>x-intercept:</u> Let y =0 and solve for x.
<u>y-intercept:</u> Let x = 0 and solve for y.
$4x + 6(0) = 12$ $4(0) + 6y = 12$
$\dfrac{4x}{4} = \dfrac{12}{4}$ $\dfrac{6y}{6} = \dfrac{12}{6}$
$x = 3$ $y = 2$
(3, 0)   (0, 2)

3. Plot the points on your graph and connect the dots.
4.Pick a test point, like (0,0) to substitute into the inequality to determine if it is TRUE OR FALSE. If TRUE, shade the region where the test point is located on the graph. If false, shade the region where the test point is NOT located on the graph.
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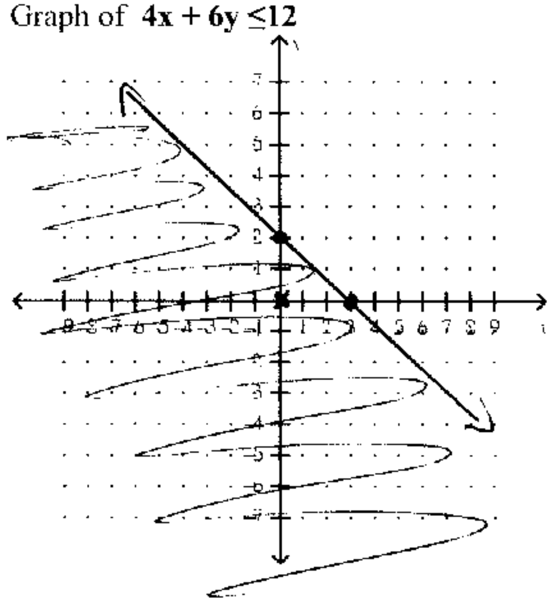
Graph of **$4x + 6y \le 12$**



test (0,0)

$4(0) + 6(0) \le 12$

$0 \le 12$

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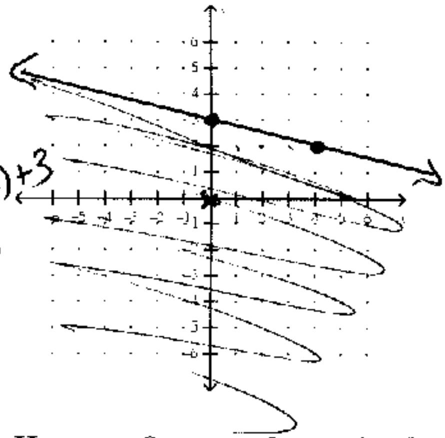
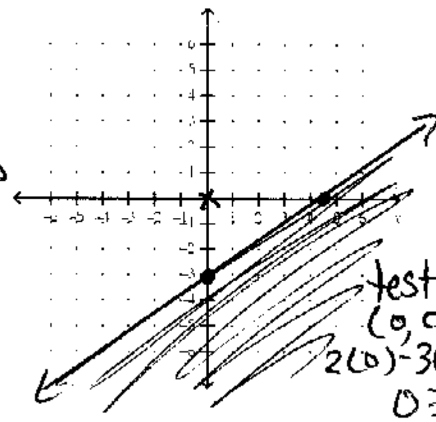
Try it: Graph the inequalities

$y \le \frac{-1}{4}x + 3$

test
(0 0)
$0 \le -\frac{1}{4}(0)+3$
$0 \le 3$
T

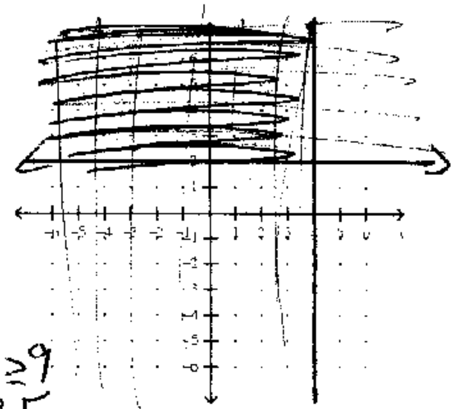


$2x - 3y \ge 9$ $\begin{cases}(0,-3)\\(4.5,0)\end{cases}$



test
(0,0)
$2(0)-3(0) \ge 9$
$0 \ge 9$ F

$y \ge 2$ and $x \le 4$



How can farmers determine how much of a crop to produce to maximize their profits and minimize costs? What do they need to know to determine this?

Linear Programming: a technique that involves maximizing or minimizing an objective function by subjecting it to constraints.

The constraints are a system of linear inequalities, which are graphed to determine a set of feasible solutions from the boundaries (vertices) of the graph. These sets of possible solutions are then entered into the objective function to determine the maximum or minimum.

Steps to solve a Linear Programming Problem:
1) Graph the region that satisfies all of the constraints.
2) Find the intersection points that bound the region, also known as the feasible solutions.
3) Evaluate the objective function at each vertex of the region.
4) Identify the solution that gives the maximum or minimum value of the function.

Example 1: Maximize profit

A fruit grower has 150 acres of land available for raising two crops, A and B. It takes 1 day to trim an acre of crop A and 2 days to trim an acre of crop B. There are 240 days per year available for trimming. It takes 0.3 day to pick an acre of crop A and 0.1 day to pick an acre of crop B. There are 30 days available for picking. The profits are \$140 per acre for crop A and \$235 per acre for crop B. Find the number of acres of crops A and B, which should be planted to maximize the grower's profit. What is the maximum profit?

x = Crop A
y = Crop B

Profit – $140x + 235y$

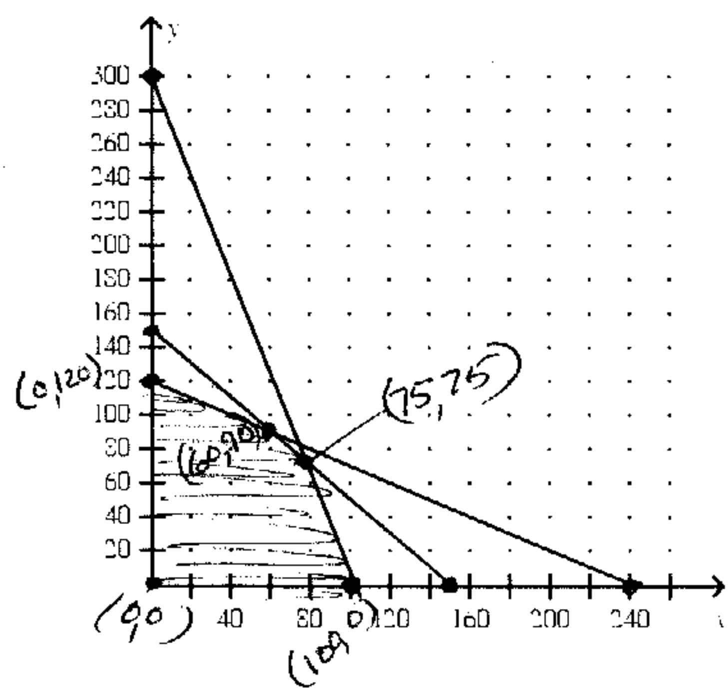
Constraints:

$x \geq 0 \quad y \geq 0$

Land $x + y \leq 150$ $(0,150)\ (150,0)$

Trim $x + 2y \leq 240$ $(0,120)\ (240,0)$

Pick $.3x + .1y \leq 30$ $(0,300)\ (100,0)$



(x, y)	$P = 140x + 235y$	
$(0,0)$	0	$140(0)+235(0$
$(0,120)$	28200	$140(0)+235(12$
* $(60,90)$	29550	$140(60)+235(9$
$(75,75)$	28125	$140(75)+235(7$
$(100,0)$	14000	$140(100)+235(0$

Max Profit of \$29,550 at 60 acres crop A, 90 acres crop B

$x + 2y = 240$
$-(x + y = 150)$
$\cancel{-x} - y = -150$
$\cancel{x} + 2y = 240$
$y = 90$
$x + 90 = 150$
$x = 60$
$(60,90)$

$.3x + .1y = 30$
$-.1\ (x + y = 150)$
$.3x + \cancel{.1y} = 30$
$-.1x - \cancel{.1y} = -15$
$.2x = 15$
$x = 75$
$75 + y = 150$
$y = 75$ $(75,75)$

Example: Minimize cost

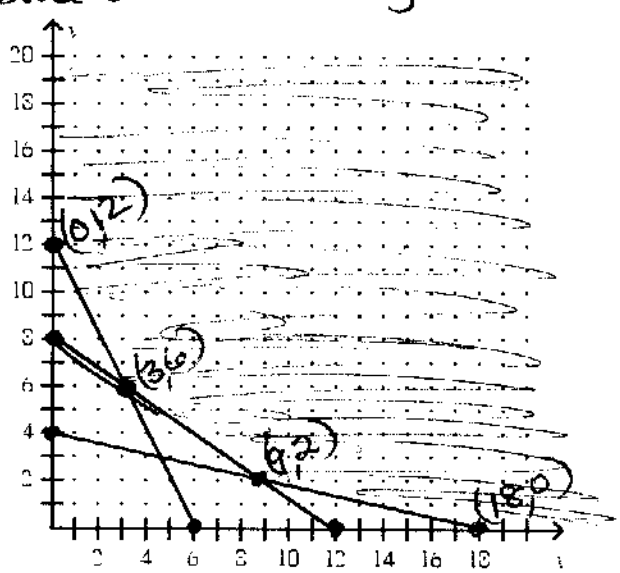
A grain co-op mixes two brands of cattle feed. Brand X costs \$25 per bag and contain 2 units of nutritional element A, 2 units of element B, and 2 units of element C. Brand Y costs \$20 per bag and contains 1 unit of nutritional element A, 9 units of element B, and 3 units of element C. The minimum requirements for nutrients A, B, and C are 12 units, 36 units, and 24 units.
Find the number of bags of each brand that should be mixed to produce a mixture having a minimum cost per bag. What is the minimum cost?

x = brand X
y = brand Y

Cost = $25x + 20y$

Constraints: $x \geq 0$
$y \geq 0$

Element A $2x + y \geq 12$ (0,12) (6,0)
Element B $2x + 9y \geq 36$ (0,4) (18,0)
Element C $2x + 3y = 24$ (0,8) (12,0)



(x y)	C = 25x + 20y	
(0,12)	240	25(0) + 20(12
* (3,6)	195	25(3) + 20(6
(9,2)	265	25(9) + 20(2
(18,0)	450	25(18) + 20(0

3 bags of Brand X
6 bags of Brand Y

\$195

Cost per bag = \$21.67

$\frac{\$195}{9 \text{ bags}}$

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Try it:
Kyla grows and sells tomatoes and green beans for a farmer's market. It costs \$1 to grow a bushel of tomatoes and \$3 to grow a bushel of green beans. It takes 1 square yard of land to grow tomatoes and 6 square yards of land to grow green beans. Kyla's budget is \$15 and she has 24 square yards of land available. If she makes \$1 profit on each bushel of tomatoes and \$4 profit on each bushel of green beans, how many bushels of each should she grow to maximize her profit? What is her maximum profit?

x = tomatoes
y = green beans

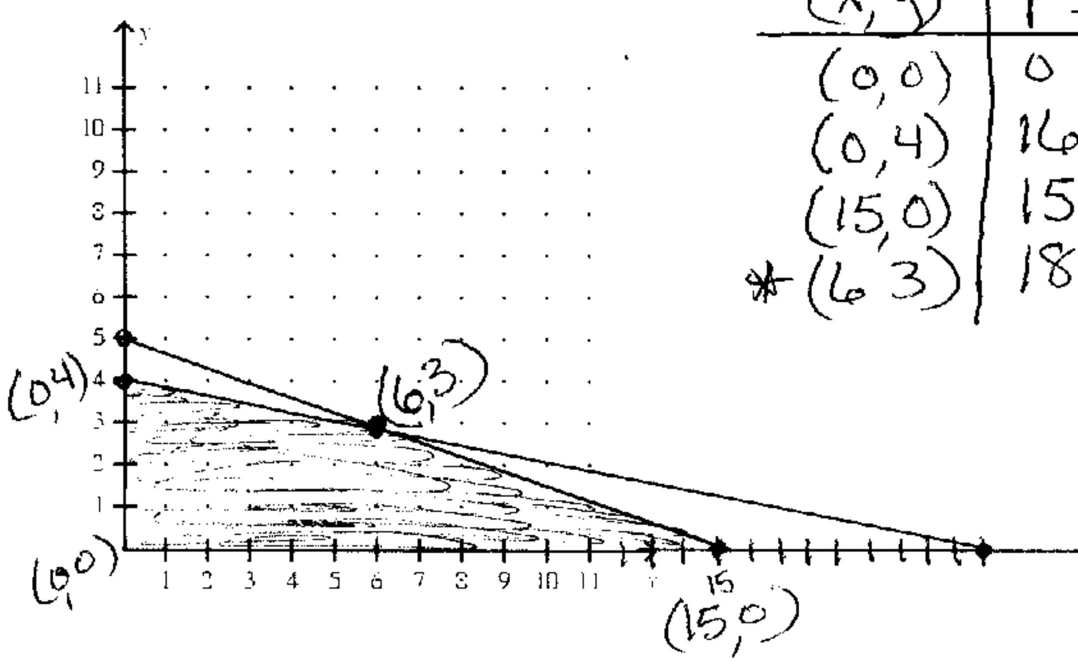
Profit = $x + 4y$

Constraints: $x \geq 0$ $y \geq 0$

Grow $x + 3y \leq 15$ (0,5) (15,0)

Land $x + 6y \leq 24$ (0,4) (24,0)

(x, y)	P = x + 4y	
(0,0)	0	0 + 4(0)
(0,4)	16	0 + 4(4)
(15,0)	15	15 + 4(0)
* (6,3)	18	6 + 4(3)



Max Profit \$18
6 bushels tomatoes
3 bushels green beans

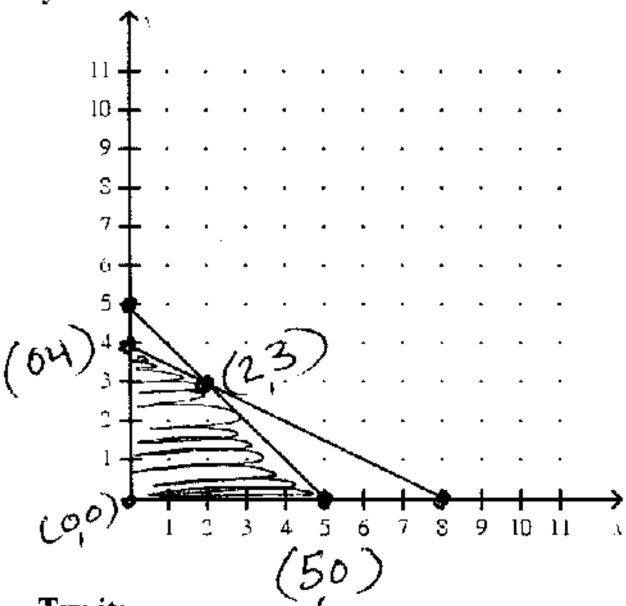
Example 3:
If profit is P = x + 3y, find the maximum profit under the following constraints:

$x + y \le 5$ (0 5) (5,0)

$x + 2y \le 8$ (0,4) (8,0)

$x \ge 0$

$y \ge 0$



(x y)	P = x+3y	
(0,0)	0	0 + 3(0)
* (0,4)	12	0 + 3(4)
(2,3)	11	2 + 3(3)
(5,0)	5	5 + 3(0)

Max Profit of 12 at (0,4)

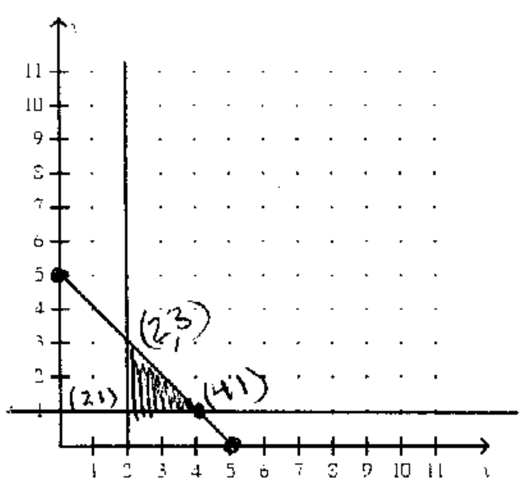
Try it:
If cost is C = 2x + 3y, find the minimum cost under the following constraints:

$x + y \le 5$ (5,0) (0 5)

$x \ge 2$

$y \ge 1$

(x y)	C = 2x+3y	
* (2,1)	7	2(2)+3(1)
(2,3)	13	2(2)+3(3)
(4,1)	11	2(4)+3(1)



Min Cost of 7 at (2,1)